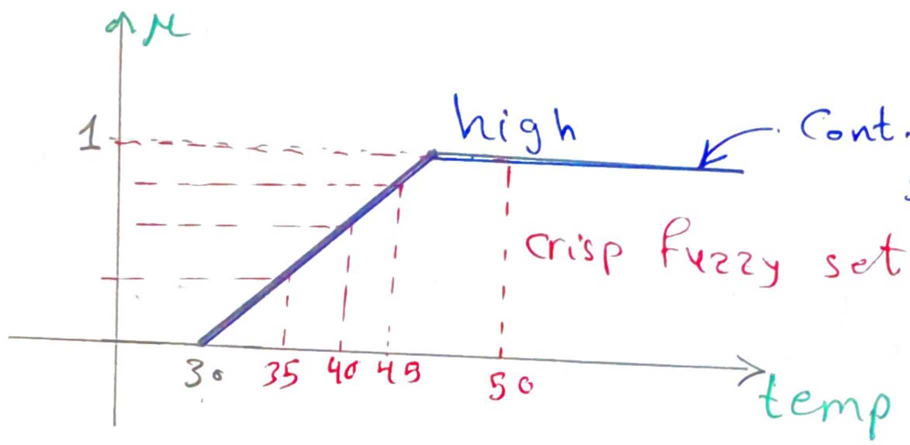
4/10/2016 الثلاثاء د. عرفة محاضرة 2

fuzzy sets :-

- ① Crisp fuzzy sets (discrete)
- ② Cont. fuzz sets



* The crisp fuzzy sets:

for any crisp fuzzy sets A, we can write the mathematical form to represent A as:

Label for fuzzy set

① $A = \{ (x_i, \mu_A(x_i)) \mid x_i \in X \}$

$i = 1, 2, \dots, n$

* $x_i \rightarrow$ the elements of the crisp fuzzy sets or the members of the crisp fuzzy sets
* $n \rightarrow$ no. of elements in the crisp fuzzy set.

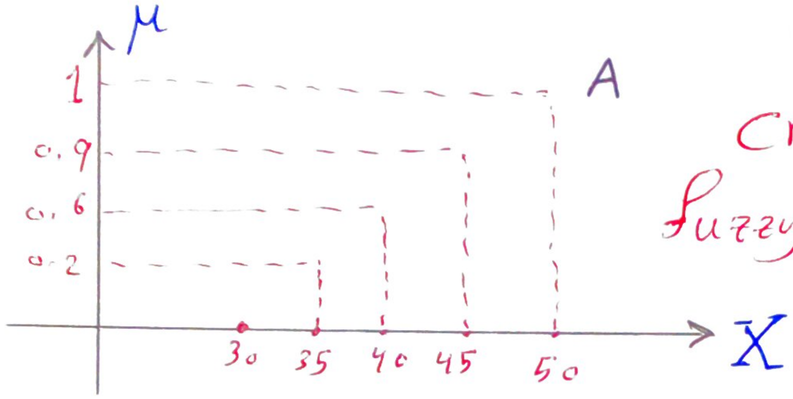
X is the universe of discourse

جميع القيم المتاحة لل fuzzy sets التي ممكن تختار منها.

Universe of discourse

The range of all possible values considered to fuzzy sets (القيم المتاحة على المحور الأفقي)

$\mu_A(x_i) \longrightarrow$ the degree or grade of the belonging x_i to fuzzy set A. ($0 \longrightarrow 1$)



Crisp fuzzy set

method $\boxed{1}$

$$A = \{(30,0),(35,0.2),(40,0.6),(45,0.9),(50,1)\}$$

element μ

method $\boxed{2}$

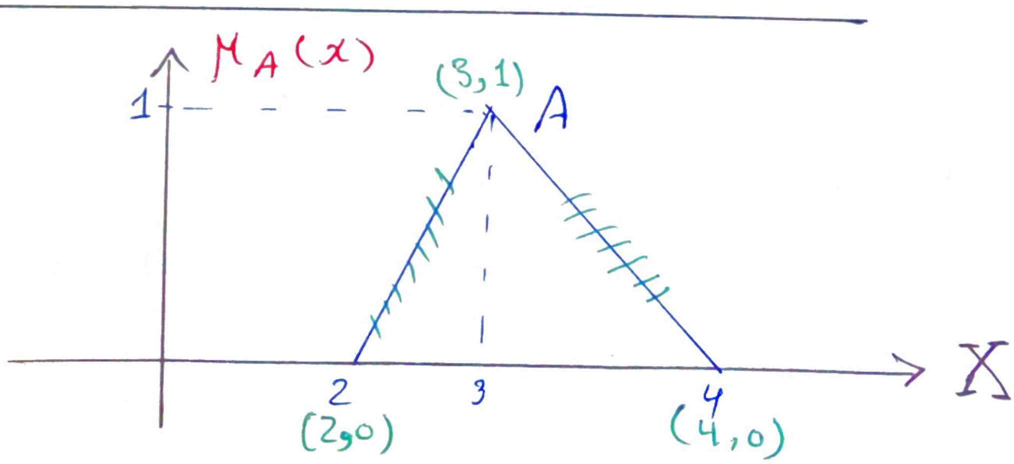
$$\left\{ \sum_{i=1}^{n} \mu_A(x_i)/x_i \;\middle|\; x_i \in X \right.$$

$$A = \{0/30 + 0.2/35 + 0.6/40 + 0.9/45 + 1/50\}$$

μ x

method $\boxed{1}$ and method $\boxed{2}$ are to describe crisp fuzzy sets

---



To describe Cont. fuzzy sets

① graphical (Previous figure)

②

$$\mu_A(x) = \begin{cases} \bigcirc & 2 \leq x \leq 3 \\ \bigcirc & 3 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow \quad \mu_A = \begin{cases} x-2 & 2 \leq x \leq 3 \\ -x+4 & 3 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

① $\frac{\mu_A - 0}{x-2} = \frac{1-0}{3-2}$

$\mu_A = x-2$

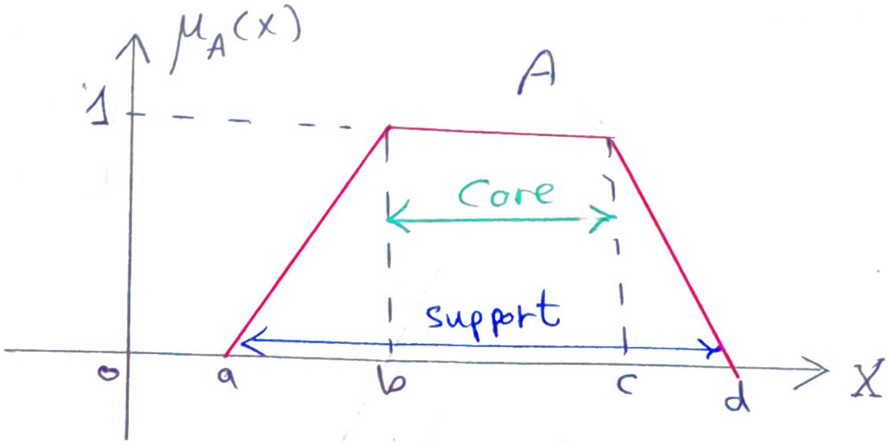
② $\frac{\mu_A - 0}{x-4} = \frac{1-0}{3-4}$

$\Rightarrow \mu_A = -x+4$

* The most common type is the cont. fuzzy sets

other concepts about fuzzy sets :-

① support



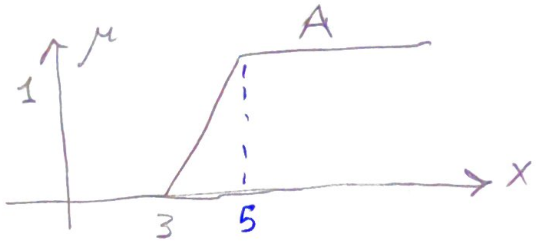
Support(A) =] a, d [

The elements (members) of fuzzy set where its

~~#~~ [membership function grade / " " degree] (MF) membership fn. degree $\neq 0$

$(\mu_A(x) \neq 0)$

ex:



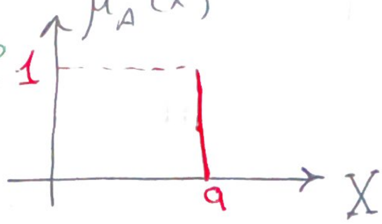
support (A) =]3 , ∞[

② core :

Core (A) = [b, c] (figure in page 3)

core (A) = [5, ∞[(figure in page 4)

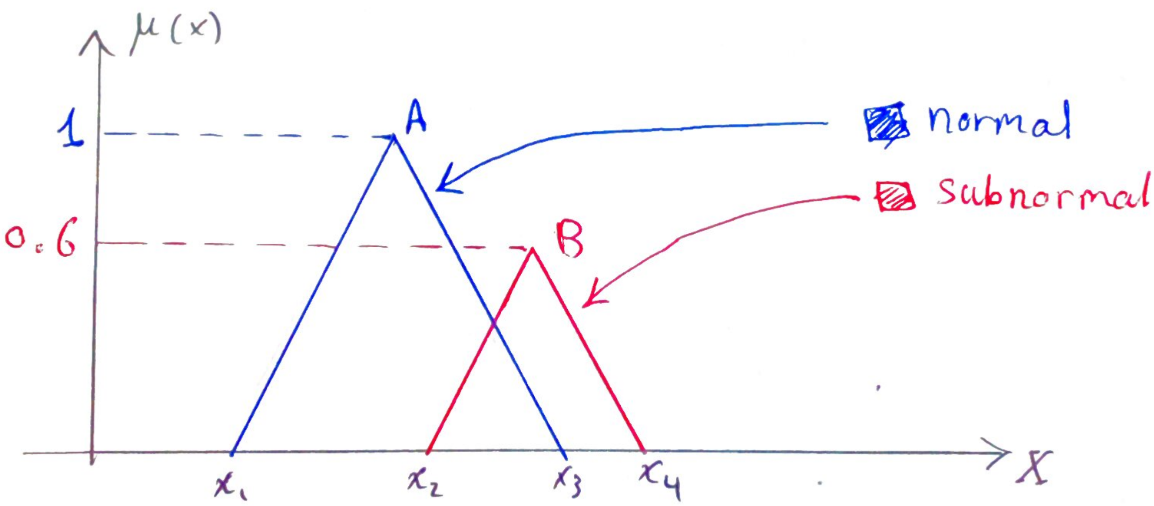
③ Single ton :



when the no. of elements = 1 with $\mu = 1$, the fuzzy set is called Single ton fuzzy set.

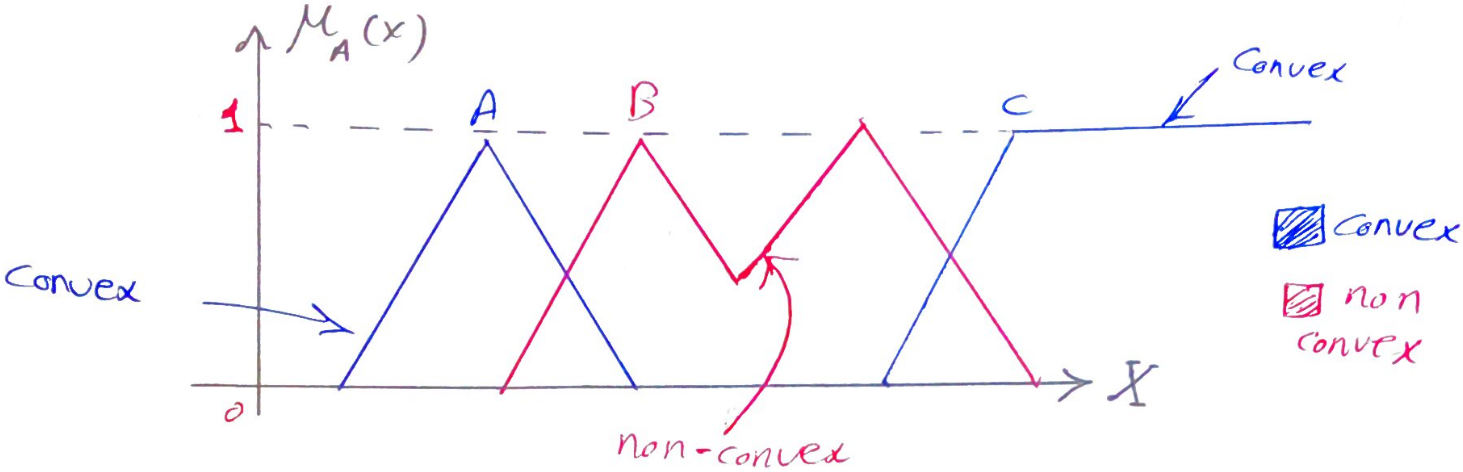
classifications of fuzzy sets

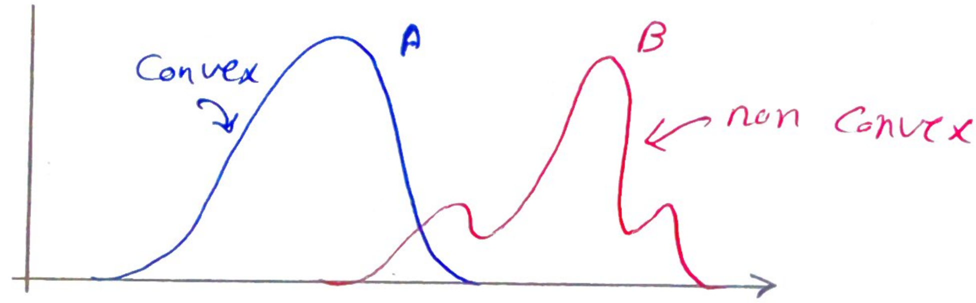
① Normal fuzzy sets and subnormal fuzzy sets



- Normal fuzzy sets contain at least one element with $\mu = 1$
- Subnormal fuzzy sets do not contain any element with $\mu = 1$

2 Convex and non convex fuzz sets:





* Convex fuzzy sets :-

The fuzzy set is convex when its μ is montonically increasing or decreasing or increasing and decreasing over the elements of the set.

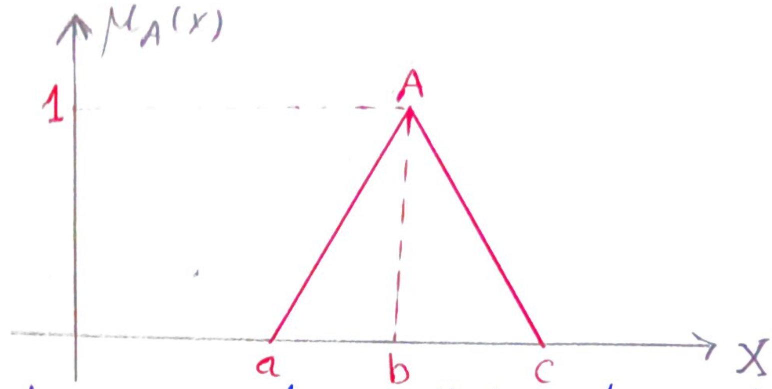
---

* In designing process of fuzzy controllers we need the fuzzy set to be:

1. normal fuzzy set
2. convex fuzzy set
3. has bounded support

Common fuzzy sets:-

[1] Triangular fuzzy set:



has 3 tuning parameters that control the shape of the fuzzy set. $a \leq b \leq c$

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{x-c}{b-c}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases}$$

① $\frac{\mu - 0}{x-a} = \frac{1}{b-a}$

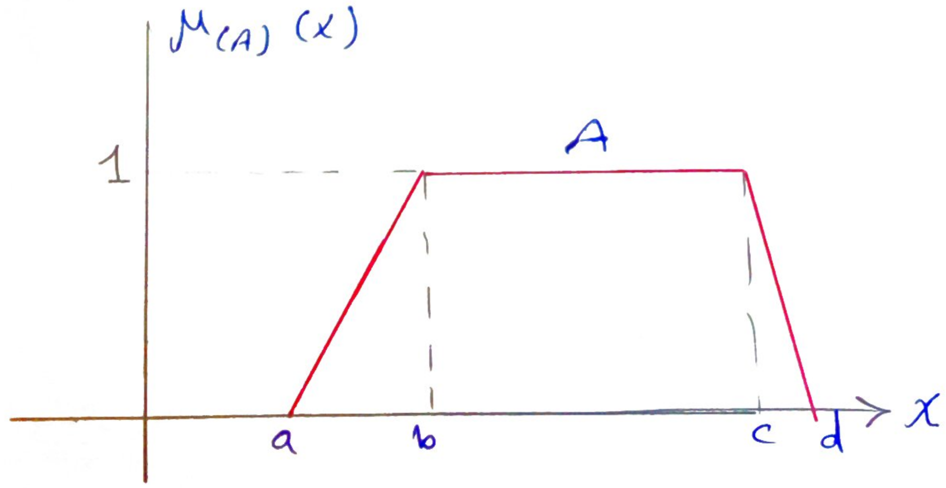
② $\frac{\mu - 0}{x-c} = \frac{1}{b-c}$

Another form:

$$\mu_A(x) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{x-c}{b-c}\right), 0\right)$$

[2] Trapezoidal fuzzy set.

has 4 tuning parameters $a \leq b < c \leq d$



$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{x-d}{c-d}, & c \le x \le d \\ 0, & \text{otherwise} \end{cases}$$

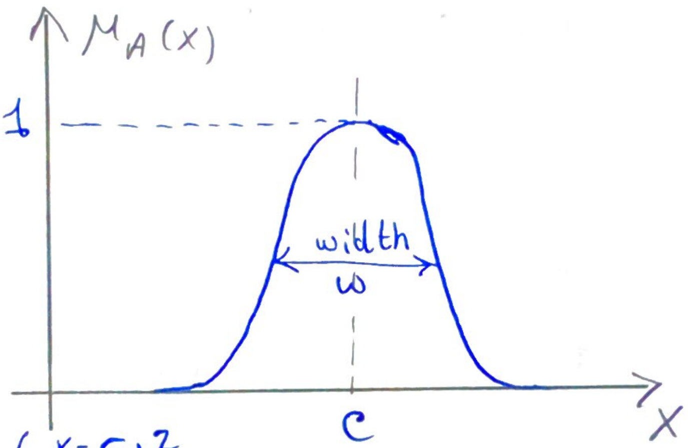
OR $$\mu_A(x) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{x-d}{c-d}\right), 0\right)$$

That form is useful for programming

[3] Gaussian fuzzy set

has 2 control Parameters "c" and "w"

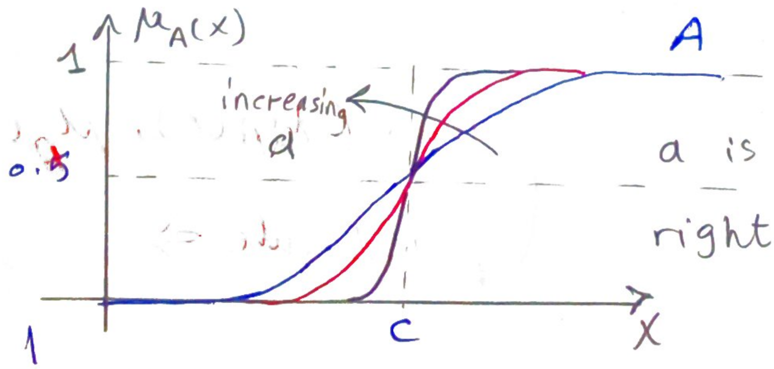
w: control width



$$\mu_A(x) = e^{-0.5\left(\frac{x-c}{w}\right)^2}$$

[4] Sigmoidal fuzzy set

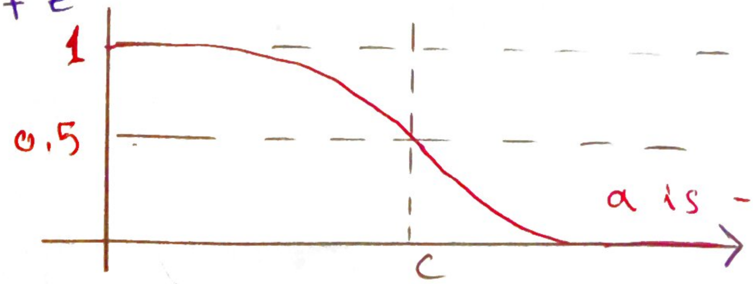
has 2 tuning Parameters "a" and "c"



a is positive right open-end

$$\mu_A(x) = \frac{1}{1 + e^{-a(x-c)}}$$

a is negative left-open end



* Sign of a determines the open-end of the shape [+ve (Right open-end) / -ve (left open-end)

Matlab fns :-

① gaussmf(x, [w c])
② trapmf(x, [a b c d])
③ trimf(x, [a b c])
④ sigmf(x, [a c])

plot(x, y)

Operations of fuzzy sets

[1] Union operation: (connective or operator)

The union of two fuzzy sets (A, B) is denoted by :

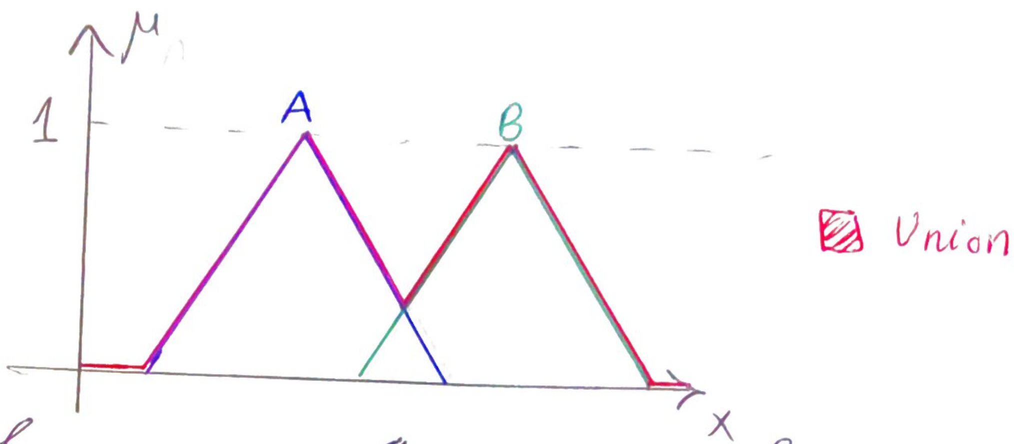
$$C = A \cup B$$

preferred

Where $\mu_A(x) = \begin{cases} \text{① max operation } \mu_c(x) = \max(\mu_A(x), \mu_B(x)) \\ \text{② product rule } \mu_c(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x) \end{cases}$

⟹ Example

ex:



ex: The fuzzy set A = "high temp" defind as:

$$A = \{ 0/36.5 + 0/37 + 0.1/37.5 + 0.5/38 + 0.8/38.5 + 1/39 + 1/39.5 + 1/40 \}$$

The fuzzy set B = "Dangerous temp" defind as:

$$B = \{ 0/37.5 + 0.1/38 + 0.2/38.5 + 0.5/39 + 0.8/39.5 + 1/40 \}$$

find $A \cup B$ = high or dangerous temp.

① $C = A \cup B = \{ 0/36.5 + 0/37 + 0.1/37.5 + 0.5/38 + 1/39 + 0.8/38.5 + 1/39.5 + 1/40 \}$

max. operation

② $C = A \cup B = \{ 0/36.5 + 0/37 + 0.1/37.5 + 0.55/38 + \dots \}$

prod. rule

لاحظ افتراق السعم

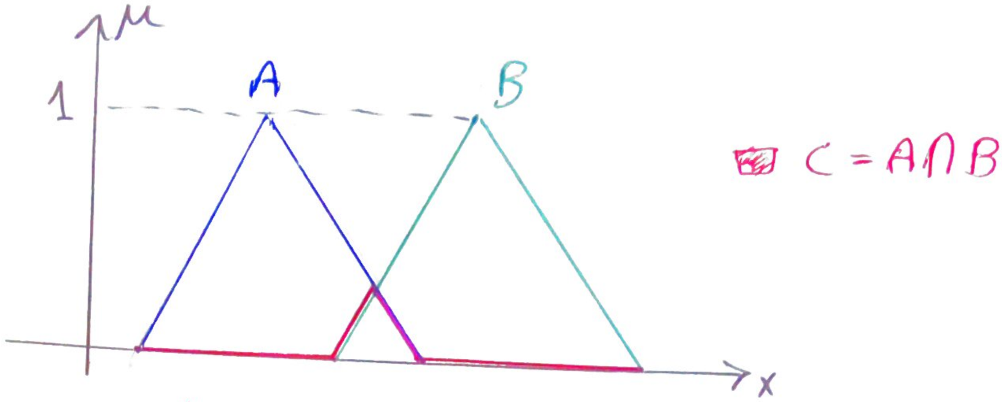
[2] intersection operation: [represent the AND connective operator]

The intersection of two fuzzy sets A and B is defined as:

$$c = A \cap B$$

minimum rule

where $\mu_c(x) = \begin{cases} \mu_c(x) = \min(\mu_A(x), \mu_B(x)) \\ \mu_c(x) = \mu_A(x) \cdot \mu_B(x) \end{cases}$ ← Product rule



3 complement operation (NOT operator)

The complement of fuzzy set يقيس عدم انتماء العنصر للمجموعة
A is denoted by $\bar{A}$ or $\sim A$ and represent to what degree the element doesn't belong to the fuzzy set.

$\bar{A}$ has $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$